

LACOSTE AND ROMBERG STABILIZED PLATFORM SHIPBOARD GRAVITY METER†

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The LaCoste and Romberg gravity meter designed for operation in gimbals was redesigned for satisfactory stabilized platform operation and a suitable stabilized platform was made. A major modification of the gravity meter was redesign of the suspension to more nearly restrict motion to a single degree of freedom. Errors due to cross coupling between horizontal and vertical accelerations and an unsuspected type of cross coupling due to gravity meter imperfections are important but correctable.

The stabilized platform gravity meter has been tested at sea. Compared to the gimbal type of gravity meter, it is simpler to operate; it will operate in rough weather; and it is considerably more accurate.

INTRODUCTION

LaCoste and Romberg has recently completed and tested a stabilized platform shipboard (or airplane) gravity meter. Earlier L & R shipboard gravity meters were hung from gimbals and corrections were made for the swinging of the gimbals. There were several reasons for changing to a stabilized platform. An important reason was the availability of inertial quality gyros with estimated lives of $1\frac{1}{2}$ years. This is a much longer life than the 1000-hour or 7-week lives given in specifications previously known to L & R.

A second reason for changing to a stabilized platform was that experience showed that it was frequently desirable to operate gravity meters at accelerations much greater than the ± 50 gal accelerations L & R gimbal gravity meters were designed for. At greater accelerations, the horizontal acceleration correction (or Browne correction) required with gimbal gravity meters becomes so large that a digital computer would be required for adequate accuracy. This would so greatly increase the cost and complexity of the gravity meter that it appeared to be necessary to use a stabilized platform.

A third reason is that inertial quality gyros have more than adequate accuracy for shipboard gravity meter operation while the overall performance of the long period pendulums used with L & R gimbal supported gravity meters was marginal. The long period pendulums also required

appreciable field testing and adjusting by operators.

Before describing the details of the new model gravity meter, the requirements for stabilized platform operation will be discussed; a comparison between these requirements and those for a gimbal gravity meter will be made; and the method of satisfying the requirements will be given.

THE STABILIZED PLATFORM

In order to determine what accuracy is required in the stabilized platform, it is necessary to find out how the gravity meter reading is affected by errors in verticality (LaCoste, 1959a). This can be done by a consideration of Figure 1, which shows a gravity meter on a stabilized platform. The sensitive axis of the gravity meter is shown to be off vertical by an angle e . The error in measuring gravity is then

$$e_g = g \cos e - a_h \sin e - g. \quad (1)$$

when g = gravity and a_h = the horizontal acceleration in the direction of the error angle e . Equation (1) can be approximated by

$$e_g \doteq -a_h e - g e^2/2. \quad (2)$$

The last term in (2) is the static error that exists even when there is no motion. For a one mgal accuracy, the second term requires a vertical accuracy of 5 minutes, which is not hard to attain.

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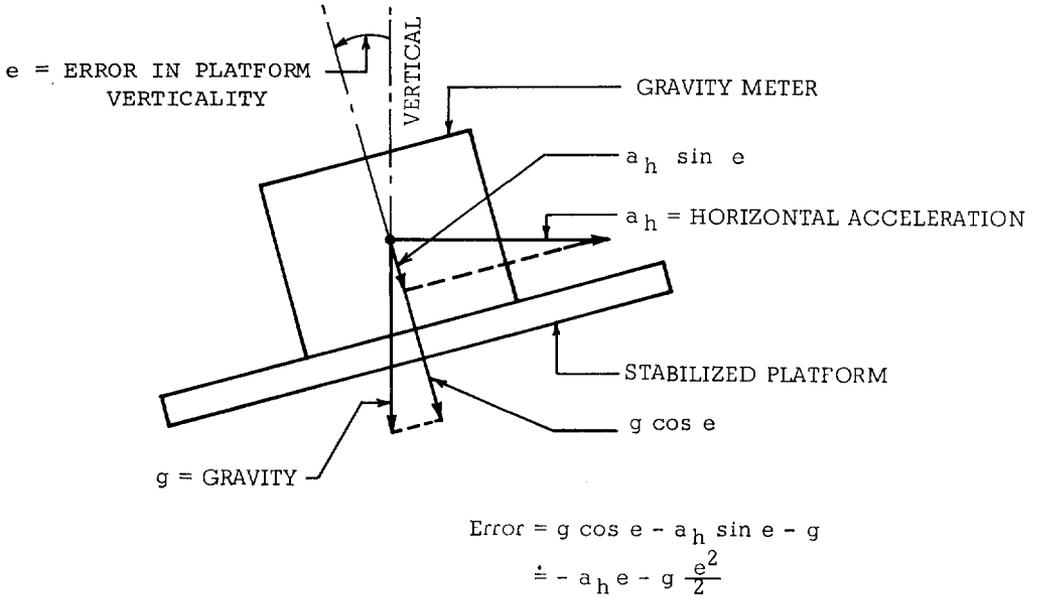


FIG. 1. Gravity meter mounted on stabilized platform.

The first term, however, depends upon the horizontal acceleration and generally requires a much higher vertical accuracy. For a one mgal accuracy when there is a sinusoidal horizontal acceleration of 0.05 g, the vertical accuracy requirement is 8 sec. However this requirement is only for errors which are in-phase with the horizontal accelerations. This is fortunate because it is difficult to achieve an accuracy of 8sec for long periods of time.

The steady state, or long-period, accuracy requirement of 5 ft is easy to check. An oil-filled level bubble can be mounted on the stabilized platform and the ship can be run in a reasonably straight line. The viscosity of the oil filters out the short-period accelerations and makes it possible to determine long term drifts in the stabilized platform. Tests of this type on the new L & R stabilized platform showed that its long term verticality errors were less than one minute, which gives a good factor of safety.

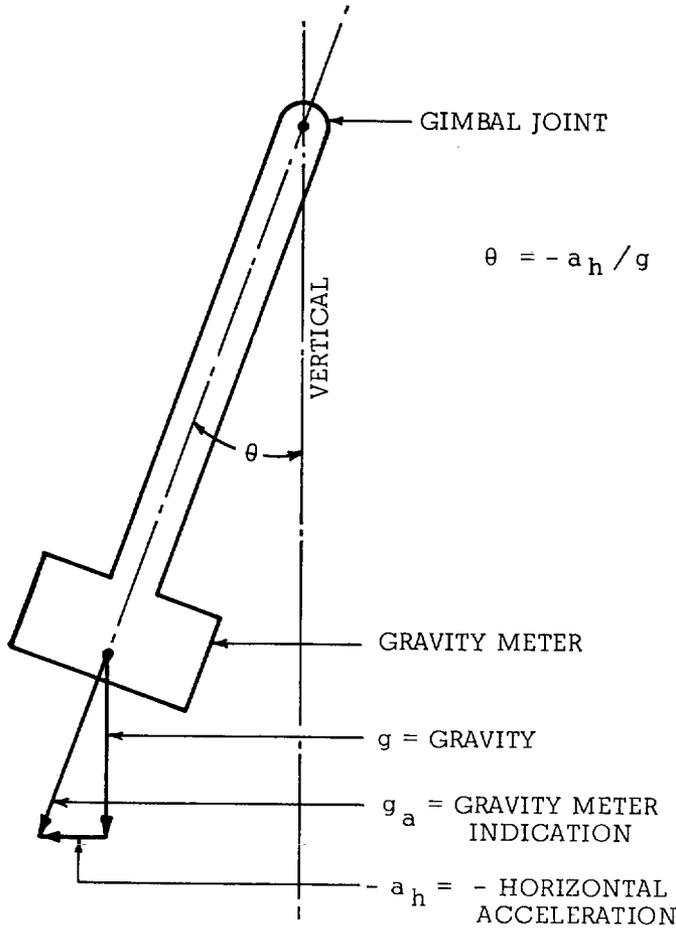
The accuracy requirement for vertical errors in phase with horizontal accelerations, however, is not easy to measure directly. A direct measurement would require a vertical reference that is known to be accurate to better than 8 sec at all times. Therefore this requirement was checked by subjecting the stabilized platform mounted gravity meter to various horizontal accelerations both in the laboratory and at sea. The accuracy of the

gravity measurements obtained showed that the required short period accuracy of the stabilized platform was realized.

In order to achieve this short period accuracy, it was necessary to do the following: (1) gyros and accelerometers of adequate accuracy were used; (2) fast response servo motors (torque motors) were used; (3) the stabilized platform was accurately balanced; and (4) the gyro erecting system, which will be described later, was given a sufficiently long period.

At this time it is worth considering the effects of vertical reference errors in a gimbal supported gravity meter because if one system has some inherent advantages over the other, such advantages should be taken into account when deciding which system to use. A diagram of a gravity meter suspended by a gimbal joint is shown in Figure 2. With such a suspension the gravity meter swings like a pendulum through an angle θ in response to the horizontal accelerations a_h . The gravity meter measures the force in the direction of its sensitive axis and the measurement is indicated by g_a .

For horizontal accelerations of periods much longer than the natural period of the gimbal suspension it is easy to determine the relation between g_a and g . In this case g_a is the vector sum of g and the negative of the horizontal acceleration $-a_h$ as shown in the vector diagram (Figure 2).



$$\theta = - a_h / g$$

$$g = g_a \cos \theta \doteq g_a - \frac{g}{2} \theta^2$$

$$g' = g_a - \frac{g}{2} (\theta - e)^2$$

$$\text{Error} = g' - g = g \theta e - g \frac{e^2}{2} \doteq - a_h e - g \frac{e^2}{2}$$

FIG. 2. Gravity meter suspended by gimbals.

Therefore

$$g = g_a \cos \theta \doteq g_a - g\theta^2/2. \quad (3)$$

The last term in (3) is known as the Browne correction or horizontal acceleration correction.

If the periods of the horizontal accelerations are not long compared to the natural period of the gimbal suspension, then (3) does not in general hold. However it can be shown to hold for all periods if the gravity meter is placed at the correct distance from the gimbal joint and if average

values are used (LaCoste, 1959b). This correct distance is the length of a simple pendulum which has the same period as the gimbal suspended gravity meter. In order to show that equation (3) holds in this case, the gimbal suspended gravity meter is treated as an undamped pendulum subjected to a sinusoidal horizontal acceleration or to a Fourier series representing an arbitrary horizontal acceleration. The approximations are also made that the average values of $\sin^2 \theta$ and $\cos^2 \theta$ are $\frac{1}{2}$ and the average values of $\sin \theta$, $\cos \theta \cos \theta$,

and $\cos \theta \sin \theta$ are 0. These are the average values for an integral number of cycles and therefore they are very close approximations for any time interval of many cycles duration.

In order to compute the Browne correction given in (3) it is necessary to measure the value of θ , and this measurement requires a stabilized reference. If there is an error e in the stabilized reference, then the measured value of θ will be $\theta - e$ and the measured value of gravity will be

$$g' = g_a - g (\theta - e)^2/2. \quad (4)$$

The error in g can then be obtained from (3) and (4); it is

$$e_g = g' - g = g\theta e - ge^2/2. \quad (5)$$

Equation (5) can be simplified by making use of the differential equation of the gimbal suspended gravity meter. It is

$$l\ddot{\theta} + g\theta = -a_h, \quad (6)$$

where l = the length of a simple pendulum having the same period as the gimbal suspended instrument. Equation (6) can be written as

$$-g\theta = a_h + l\ddot{\theta}. \quad (7)$$

The right side of (7) however is the horizontal acceleration a'_h at the gravity meter which is a distance l below the gimbal joint. Therefore (5) becomes

$$e_g = -a'_h e - (ge^2)/2. \quad (8)$$

If equations (8) and (2) are compared and it is noted that the acceleration at the gravity meter in (2) is denoted by a_h and in (8) it is denoted by a'_h , it can be seen that the two equations are identical. The gimbal supported gravity meter is therefore just as sensitive to errors in its stabilized reference as the stabilized platform mounted gravity meter is. There is no preference in this respect. Also it is obvious that gyros and accelerometers could be substituted for the long period pendulums in the gimbal system, which would result in an improvement in performance.

In order to attain the vertical accuracy required by equation (2), a satisfactory gyro erection system had to be designed for the stabilized platform. A diagram of the erection system is included in the block diagram of Figure 3. There are two gyros, only one of which is shown. Each gyro

controls its torque motor to make the stabilized platform follow the gyro. This however, is not sufficient to insure verticality of a reference line on the platform because: (1) the reference line might not be vertical to begin with; (2) gyros have some drift; and (3) the earth rotates and gyros tend to remain fixed in space. In order to attain verticality, accelerometers or levels are mounted on the platform as shown. If the accelerometers do not indicate level, they put out error signals. Each error signal plus a constant times its integral is fed to the corresponding gyro to gradually precess it to bring the reference line on the platform to vertical.

One of the main reasons for using an integral term in the feedback from the accelerometers is that it eliminates an error that would otherwise be present because of the rotation of the earth. The earth's rotation requires an equal precession rate of the gyro in order to keep the platform vertical. This precession rate would require a constant error signal from the accelerometer if there were no integral term in the feedback.

The use of both error and integral of error in the feedback gives a second order differential equation, and therefore the stabilized platform behaves exactly like a long period damped pendulum. The amount of integral feedback determines the natural period of oscillation of the stabilized platform and the amount of ordinary feedback determines its damping.

The L & R gimbal supported shipboard gravity meters use long-period pendulums as vertical references; so again it is found that there is mathematical equivalence between the old gimbal and new stabilized platform meters. In the old models satisfactory results were obtained with pendulums of about two minute periods. If the periods are too short, errors are introduced because of fishtailing of the ship and because of long period wave motions. If the periods are too long, the long-period pendulums tend to drift, and transients introduced after a ship turn became objectionable.

Because of the greater stability of gyros as compared to the long-period pendulums, it is possible to use periods much longer than the previously used 2-minute periods. It is also possible to reduce the time of objectionable transients after a ship turn by providing slewing switches to manually bring the platform to approximate vertical after the turn. Such slewing switches are provided in the new model and periods of either 4 or 6 min-

gimbal suspension itself filters out horizontal vibrations and there is no problem in adding a spring or shockcords to filter out vertical vibrations. However shockmounting a stabilized platform causes some difficulties because the servo motors controlling the platform need to be mounted on a firm base in order to act quickly. A shockmounted base makes them act sluggishly and causes them to hunt.

In order to overcome this problem, parallel linkages were installed between the shockmounted base and the supporting frame fixed to the deck of the ship. These linkages are shown in the photograph of the stabilized platform gravity meter, Figure 4. The linkages allow translation of the shockmounted base in any direction but prevent rotation of it relative to the ship. The base therefore provides a good support for the servo motors.

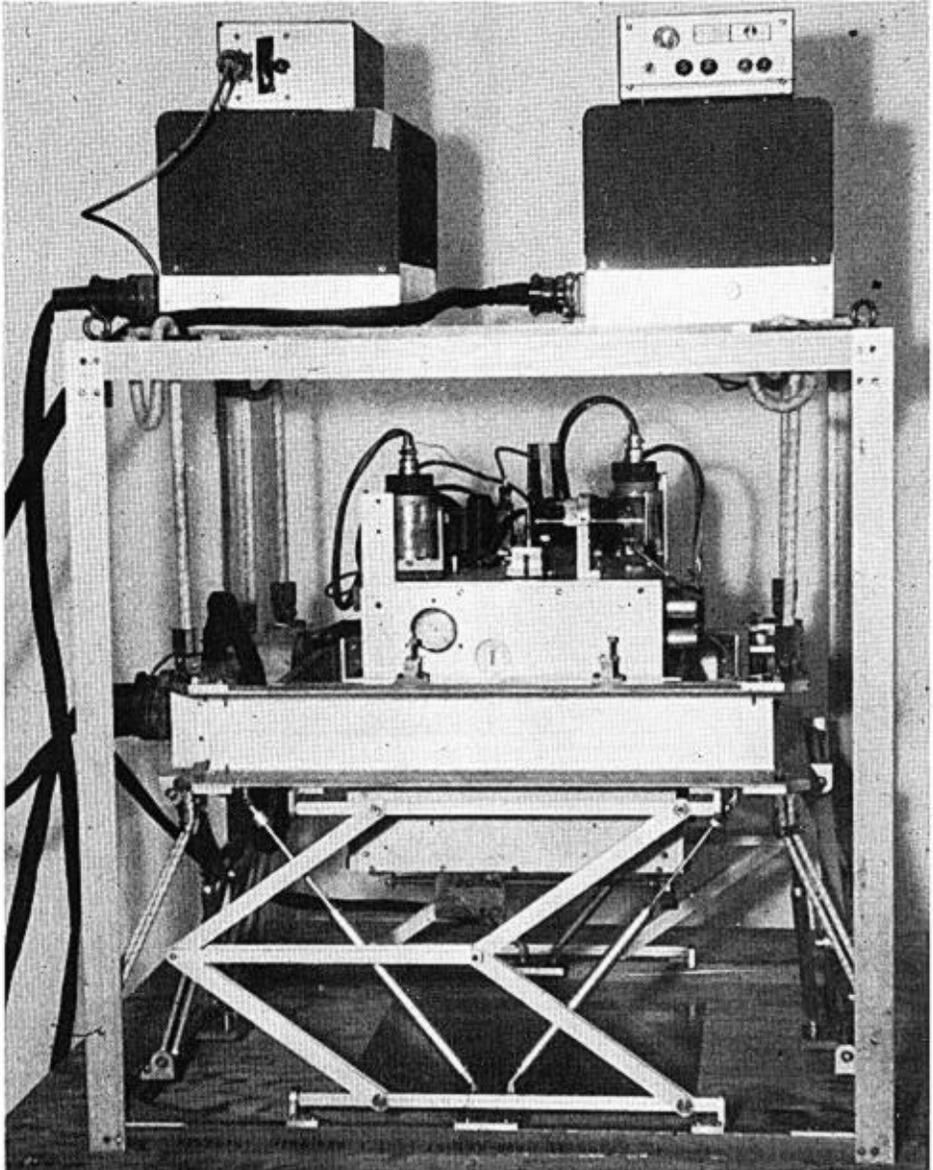


FIG. 4. Stabilized platform gravity meter.

The range of motion of the stabilized platform was made ± 30 degrees, which is probably more than adequate.

The gyros used are Honeywell model GG49, which are of inertial guidance quality. The accelerometers are Donner model 4310. The accelerometers appear to have adequate accuracy, but more accurate ones are available for use if it becomes desirable. The stabilized platform characteristics are summarized in Table 1.

THE GRAVITY METER

The first tests made with the new L & R stabilized platform were made with an L & R gravity meter that had previously been used with gimbals, although it was realized that the requirements for a stabilized platform gravity meter were not the same as for a gravity meter suspended in gimbals. The basic principles of the L & R gravity meter are given in U. S. patent 2,977,799.

The main difference in requirements is that a stabilized platform gravity meter must withstand much greater forces normal to its sensitive axis. Since it is held vertical at all times, all forces produced in it by horizontal accelerations are normal to its sensitive axis. On the other hand when a gravity meter is supported at the correct distance below a gimbal joint, all forces in it are directed along its sensitive axis regardless of how it is accelerated (LaCoste, 1959b). The truth of this statement for long periods is obvious from the vector diagram in Figure 2. The proof for all periods is given in the reference just cited. Because of this absence of lateral forces with a gimbal suspension, all L & R gimbal gravity meters were made only stiff enough to withstand about 0.07g horizontal accelerations before there was interference between the fixed and movable parts of the gravity meter.

A second difficulty in operating a gravity meter on a stabilized platform is the problem of cross coupling error (LaCoste and Harrison, 1961) due to interaction between horizontal and vertical accelerations. Cross coupling errors are not present in gimbal supported gravity meters when the gravity meter is at the correct distance below the gimbal joint. More will be said about cross coupling later.

In spite of these known difficulties, tests were made by Dr. J. C. Harrison of Hughes Research Laboratories with an unmodified L & R gravity meter on the new stabilized platform. The results

Table 1. Main features of new L & R stabilized platform

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|---------------------------------------------------------------------------|
| 1. Inertial quality gyros with 1½ years life. |
| 2. High response torque motors. |
| 3. Stabilized platform periods of 4 and 6 minutes (easily increased). |
| 4. Long term accuracy of 1 minute. |
| 5. Short term accuracy of 8 sec in-phase with horizontal accelerations. |
| 6. Shock mounted to attenuate vibrations with periods shorter than ½ sec. |
| 7. Angular range of ± 30 degrees. |
| 8. Accelerometer range of 0.2g (easily increased). |
| 9. Slewing switch for rapidly leveling platform after a turn. |
| 10. All solid state electronics. |

at accelerations greater than about 0.05g were of course bad, as was expected. The results at accelerations below 0.05g were comparable to those obtained with a gimbal supported gravity meter but probably not quite as good. A cross coupling computer had been built, but it had electrical troubles during the test. Cross coupling errors were therefore certainly present. The stabilized platform operated very well.

As a result of the test a new gravity meter was designed to meet the requirements for operation on a stabilized platform. An important change in the new model was to redesign the movable beam and its hinge so as to reduce the translational yielding of the beam resulting from the accelerations to which the gravity meter is subjected. This stiffening was done without appreciably affecting the freedom of rotation of the hinge, and therefore without appreciably affecting the sensitivity of the gravity meter. The new model was made 40 times stiffer than the earlier model.

A second important improvement was to reduce the error caused by vertical accelerations. Since vertical accelerations have to average out to zero on a ship, their average effect on a perfectly linear gravity meter would also average out to zero. However, if there is any nonlinearity in the gravity meter, it will produce an error that is generally proportional to the square of the vertical acceleration. The nonlinearity in the new model gravity meter was reduced to such an extent that the gravity meter error was well under one mgal for vertical accelerations of 0.1g, which is the limit of the present L & R vertical testing machine. This was done partly by some refinements in design and partly by careful adjustments. The design changes also made the gravity meter capable of

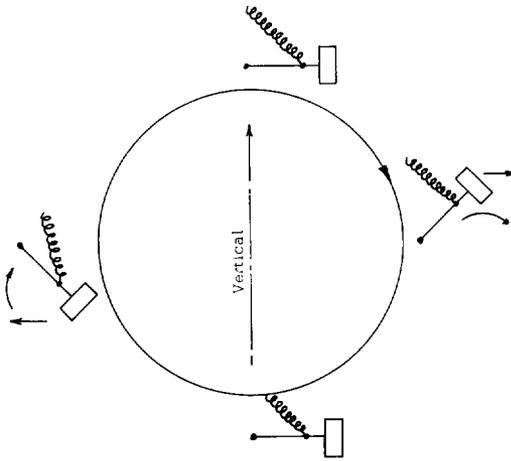


FIG. 5. Diagram showing inherent type of cross coupling.

withstanding rougher handling without getting out of adjustment.

A third improvement in the new gravity meter was to increase its damping to such an extent that it can withstand vertical accelerations of $\pm 0.5g$ at a period of 7 sec without having interference between fixed and movable parts of the gravity meter. At a period of 3.5 sec, the gravity meter will withstand vertical accelerations of $\pm 1g$. The damping was increased by a factor of 2 or 3 over that of the earlier model.

The increased damping also decreased the ordinary cross coupling effect by the same factor because it reduced the motion of the beam by that factor.

A fourth improvement was design and adjustment changes to reduce certain types of cross coupling error that have not been considered to date. In order to understand what was done, a discussion of the cross coupling problem will be given.

One type of cross coupling has been known (LaCoste and Harrison, 1961) for several years. Although it has been described in the literature it is probably worthwhile to give an example of how it can cause an error in a gravity meter. Figure 5 is a diagram of a gravity meter being moved in a circular path in a vertical plane. If the gravity meter is highly damped, its beam deflection will be proportional to the negative of the vertical velocity to which the gravity meter is subjected. The positions of the gravity meter beam will therefore be as shown in the figure. The beam is

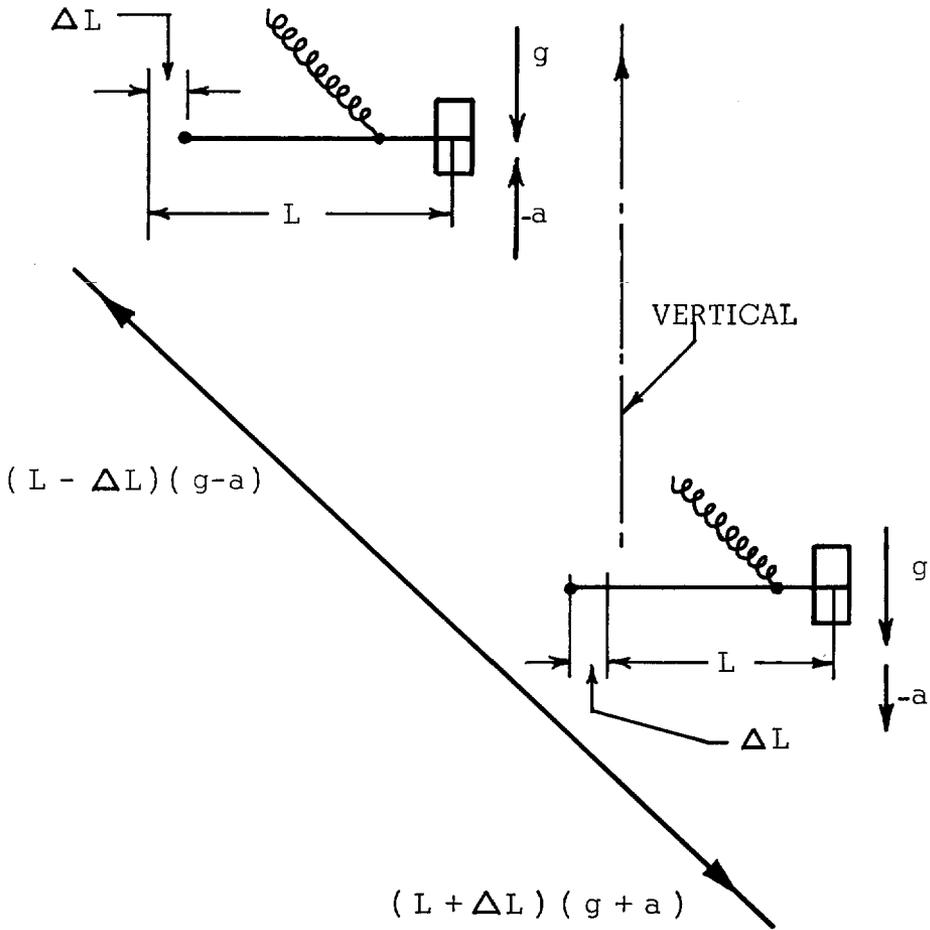
assumed to be horizontal at the top and bottom where the vertical velocity is zero.

The effect of the horizontal accelerations will now be considered. At the right there is a centrifugal force to the right, and since the gravity meter weight is above center, this force will produce a clockwise torque. At the left the centrifugal force is to the left; but the weight is below center, and therefore the torque is again clockwise. Consequently the torque does not average out but makes the beam move in a clockwise direction. If the direction of the circular motion is reversed, the error will also be reversed.

This cross coupling error is present even if the gravity meter is not overdamped, but in this case the error occurs with a ramp motion rather than with a circular motion. The error can be eliminated by keeping the beam accurately nulled or by using a gravity meter that has symmetry about a vertical axis; otherwise the error is inherent in a stabilized platform gravity meter. A cross coupling computer was therefore provided with the new model gravity meter; it is shown in the block diagram of Figure 3. It receives inputs of beam position and horizontal acceleration along the beam and it computes their product, which is a measure of the required inherent cross coupling correction. The output appears on the recorder so that it can be corrected for. The correction can also be made automatically to the gravity reading in the automatic reader if desired.

It is interesting to note that a gimbal supported gravity meter is not subject to cross coupling errors if the gravity meter is placed at the correct distance below the gimbal joint. This "correct distance" is the same distance that makes equation (3) correct for all periods. This adjustment avoids cross coupling errors by eliminating all forces on the gravity meter except a force along its sensitive axis (LaCoste, 1959b). The absence of cross coupling effects in gimbal supported gravity meters is a real advantage over stabilized platform gravity meters, but the cross coupling effect is usually less than 20 mg and therefore a simple cross coupling computer can make adequate corrections.

The previously described cross coupling effect will be called the inherent type, because it is inherent in certain types of gravity meters. The types of cross coupling about to be described are due to imperfections in the gravity meter and



Average Measured Gravity

$$= (1/2)(L - \Delta L)(g - a) + (L + \Delta L)(g + a)$$

$$= Lg + \Delta L a$$

FIG. 6. Diagram showing an imperfection type of cross coupling.

therefore will be called imperfection types. They often have even greater magnitudes than the inherent type. On the other hand it is possible to make the imperfection type negligible by careful design and adjustment of the gravity meter. There are many ways in which imperfections can cause cross coupling errors, and it is often difficult to determine which is the major cause of error. In order to show how they can cause errors, two examples will be given.

Figure 6 shows a gravity meter with a hinged

beam being moved up and down a ramp. There will always be some elasticity in the beam and in the wires providing a hinge for it. This elasticity will allow the center of gravity of the beam to move horizontally in response to horizontal accelerations, thereby changing the moment arm of the beam. The moment arm on the right of the figure can be denoted by $L + \Delta L$ and on the left it can be denoted by $L - \Delta L$. Also because of vertical acceleration the vertical force on the beam at the right will be $g + a$, where a is the vertical accelera-

tion necessary to reverse the motion. Similarly on the left the force will be $g-a$. The average of the two moments will be $Lg+a\Delta L$ as shown. However, the average moment would have been Lg if there had been no motion. Therefore the term $a\Delta L$ shows a cross coupling error.

The previous considerations show how important it is to make the gravity meter rigid with respect to translational yielding. However, even if the gravity meter is made very stiff, the center of gravity of the spring will shift in response to horizontal accelerations, and such a shift can have the same effect as a shift in the center of gravity of the beam.

Another example of cross coupling is shown in Figure 7, which is a top view of the gravity meter. The spring supporting the beam is shown at the right, and the wires providing a hinge are shown at the left. The hinge axis is labeled. In order to restrict horizontal translation along the axis of rotation, the two wires shown are used. The upper wire is shown in the axis of rotation and gives no trouble with cross coupling. The lower wire, however, is shown to the right of the axis of rotation and will give cross coupling errors, as can be seen from the following considerations.

The effect of vertical accelerations will be considered first. It will be assumed that the beam will yield to some extent in response to such accelerations. The point of attachment of the lower (off axis) wire to the beam will then move up and down. When it moves down, the wire will tend to raise the weight and vice-versa. Since the upward and downward accelerations will average out to zero, there will be no net effect.

However if horizontal accelerations are present as well as vertical, there can be errors. The horizontal accelerations will have the effect of tightening or loosening the wire, and if the wire is always tight when it is pulling up and always loose when it is pulling down, then there will be a net upward pull on the weight. This gives a cross coupling error.

In order to make negligible the errors due to this imperfection type of cross coupling, it was found necessary to make tests with various types of motion as well as making some design changes and some careful adjustments.

A list of the specifications which the new model gravity meter fulfills is given in Table 2.

The block diagram of Figure 3 has been partly explained already. The operation of the remaining

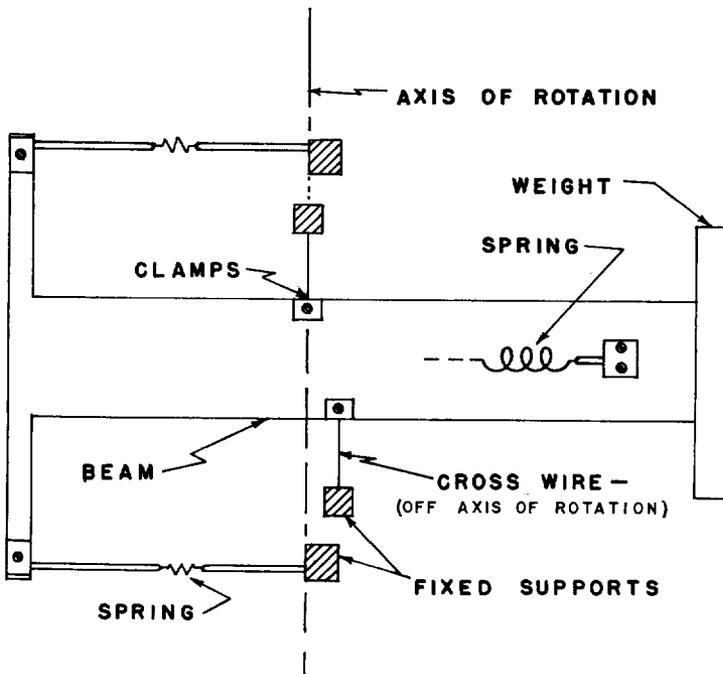


FIG. 7. Diagram showing a second imperfection type of cross coupling.

Table 2. Main features of new L & R shipboard gravity meter unit

1. Reduced yielding to lateral accelerations by a factor of 40 compared to earlier model.
2. Linearity improved to reduce vertical acceleration errors to less than one mgal at vertical accelerations of $\pm 0.1g$.
3. Damping increased to permit operation at $\pm 0.5g$ at a 7 sec period without interference between fixed and movable parts.
4. Cross coupling computer for correcting inherent type of cross coupling.
5. Imperfection type of cross coupling made negligible by careful design and adjustment.
6. Simpler to operate and maintain than earlier model

Note: The accuracy at sea is discussed in the following article by Nettleton and LaFehr.

parts is as follows. A servo in the automatic reader controls the spring tension S to approximately null the beam position B of the gravity meter. The servo is made slow so that it will not follow the wave accelerations very much. Gravity is computed in the automatic reader from the spring tension and the derivative of the beam position after some filtering. Gravity is then recorded on a strip chart recorder or tape recorder or both. For

monitoring, strip chart records are made of the horizontal accelerations and the beam position both with and without filtering.

Tests of the new L & R stabilized platform gravity meter have been made by Geophysical Associates International, by the U. S. Naval Oceanographic Office and by Dr. Roland von Huene of the Naval Ordnance Test Station at China Lake, California. The results of the tests made by Geophysical Associates International appear in the article following this article. It appears that the new model gives considerably better accuracy than the old model in addition to being able to operate in much rougher weather. It is also very much simpler to operate and to keep in adjustment.

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