CROSSCORRELATION METHOD FOR EVALUATING AND CORRECTING SHIPBOARD GRAVITY DATA†

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In shipboard gravity meter operation it is important to be able to evaluate the data and to make corrections if the gravity meter is not performing perfectly. These objectives can be achieved to some extent by (1) describing the ship motion with the terms of a power series in ship accelerations and velocities, and (2) using the crosscorrelations between these terms and observed gravity to evaluate performance and to obtain a corrected gravity that does not correlate with any of the power series terms. The method makes use of the simultaneity of the variations of observed gravity with those of the power series terms.

The method is applied to some actual gravity data taken with a gravity meter that was not performing properly. The results show clearly the poor performance and indicate the cause. Computed corrections greatly reduce the noise in the results and give a systematic correction. Tests of the gravity meter after it was returned for repair confirm the results of the crosscorrelation analysis and show that the computed corrections are accurate.

INTRODUCTION

Several principles have been applied to processing shipboard gravity data. A principle that has been universally used is to filter out periods due to ocean wave motion. These periods are short enough to be removed without losing data of geophysical significance. Filtering at somewhat longer periods has also been used to smooth the data even though such filtering often loses some possibly useful geophysical detail. In some cases specially designed filters have been used to remove noise that is characteristic of the ship motion. In addition to the above methods for smoothing data, statistical methods have been used to correct for systematic errors by making comparisons at line crossings (LaFehr and Nettleton, 1967; Foster et al, 1970).

The crosscorrelation method described in this article not only smooths the data but also gives a correction for systematic errors. The basic principles of the method were suggested by the writer (LaCoste, 1967). The principles are: (1) There should be no crosscorrelation between variations in ship motions and variations in observed gravity corrected for Eötvös effects. (2) The ship motions can be described by several monitors, which consist of ship accelerations and velocities and products of two or more such accelerations and velocities. (3) Because of (1) and (2) there should be no crosscorrelation between variations in observed gravity and variations in any of the monitors. (4) If there are any such crosscorrelations, observed gravity can be corrected by adding to it whatever fractions of the monitors are required to give the zero correlations.

It will be noted that the method makes use of no data except outputs of the gravity meter and stabilized platform and the Eötvös effect; and it has been found that good results can be obtained even if the Eötvös effect is not used. The method is therefore of great value in determining how well the gravity meter is performing during the course of a survey. In particular, it can show whether noisy data are due to erratic ship steering or poor

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gravity meter performance. If the gravity meter is performing badly, the crosscorrelation method can often pinpoint the cause and give accurate corrections to be added to observed gravity. How accurate the corrections are depends on how erratic the gravity meter malfunctions are; errors caused by such things as a bad stabilized platform bearing are too erratic to allow very good corrections to be made. Obviously the method is of great value to the gravity meter manufacturer as a guide toward product improvement.

After giving the details of the method, the results of applying it to some actual data are shown. The data was made available by R. C. Couch and M. Gemperle of Oregon State University. It is obvious that the crosscorrelation method can be used in addition to the other methods previously mentioned.

**QUALITATIVE CONSIDERATIONS**

The validity of assumption (1) above is obvious. Assumption (2) is valid if enough monitors are used because the terms in a power series are a complete set of functions for describing ship translations and because angular motions of the ship need not be taken into account if good servos are used on the stabilized platform supporting the gravity meter.

A condition the assumptions have to satisfy is that the correction to observed gravity on a stationary ship be zero. Since, according to (4), the correction is a linear combination of the monitors, the condition will be met if all the monitors are zero for no ship motion. It will be noted that the monitors have been chosen in (2) to satisfy that requirement.

It is obvious that suitable monitors must be chosen if the method is to give good results. However, if irrelevant monitors are chosen, errors will not be introduced. The irrelevant monitors merely do not affect the results. The monitors can be chosen by trial and error. The ones we have chosen in this article are appropriate for LaCoste and Romberg gravity meters. Adding the Eötvös correction as a monitor would probably be worthwhile. It should be pointed out that most of the chosen monitors apply to the usual stabilized platform malfunctions as well as to gravity meter malfunctions.

Assumption (3) follows from (1) and (2), but some justification for assumption (4) is in order. Nearly all instruments give better accuracy under static conditions than under dynamic conditions; and gravity meters are no exception. It therefore follows that if the amplitude of ship motion varies along a course, the observed gravity meter readings will be more nearly correct when the amplitude is small. This suggests that an extrapolation be made for zero ship motion. The extrapolation can be made by the following two steps: (a) finding a linear combination of the monitors that best fits the shape of the observed gravity curve, and (b) subtracting this linear combination from the observed gravity. These two steps are equivalent to assumption (4).

It should be noted that the extrapolation or correction method just described not only makes the gravity trace smoother but also offsets the trace, i.e., makes a systematic correction. It is often possible to get an independent check of this offset from line crossings. The discrepancies in observed gravity at line crossings should be proportional to differences in the monitor at line crossings. The factor of proportionality should be the same as that determined by the crosscorrelation method. This has been the case in the few examples that have come to the attention of the writer.

An inspection of Figure 1 will illustrate some of the things that have been discussed and will bring out some additional points. In the figure, observed gravity and one of the monitors are plotted against time for a gravity meter that is not operating properly. It is seen that there is strong positive correlation between gravity and the monitor. If this correlation exists for extended periods of time there is reason to believe not only that the gravity meter is performing poorly but also that the errors can be reduced by subtracting a fraction of the monitor from observed gravity.

In Figure 1, the crosscorrelation is apparent because the anomalies in each trace occur at the same time, and the ratio of their magnitudes is substantially constant. Since this visual crosscorrelation depends only on visual anomalies in the traces, some measure of the anomalies should be used in a quantitative determination of crosscorrelation. The chosen measure of the anomalies is the second time derivative, because it is proportional to the curvature of the traces.

Another question that arises in connection with estimating the crosscorrelation is the length (time duration) of the anomalies that are considered significant. Anomalies long enough to be mistaken
for legitimate gravity anomalies should not be given much weight. The same is true of anomalies having the same periods as ocean waves. Based on visual crosscorrelation, a reasonable range for the periods of significant anomalies appears to be between 1 and 15 minutes, which corresponds to periods of about 2 to 30 minutes. Accordingly, in the quantitative treatment, the second derivatives of the monitors and observed gravity are filtered enough to give significant weight only to the periods of the above range. The filter chosen for the present analysis has the response shown in Figure 2. It was found that the results of the analysis were scarcely affected by using filters that changed the period of maximum response by ±50 percent.

As previously mentioned in connection with Figure 1, the anomalies in the monitor and in observed gravity are closely in phase. This is so because the response time of the gravity meter (without filtering) is negligible compared to the times involved, and because the monitors and gravity were filtered the same way. It therefore appears that crosscorrelations need be considered only for zero time differences between the variables. This simplification saves considerable processing time.

DETAILS OF COMPUTATION

In processing the data it is advantageous and often necessary to filter the data before differentiating it to obtain curvatures. LaCoste and Romberg shipboard gravity data are ordinarily filtered before being recorded on magnetic tape with three stages of low-pass resistance-capacitance filtering with a time constant of 20 sec per stage. (A single stage of such filtering is merely a resistor and capacitor in series. The input is the voltage across both elements, and the output is the voltage across the capacitor.) With this initial filtering, sampling rates of 10 and 30 sec gave almost identical results, which indicates that a 30-sec sampling time is short enough. If it is desirable to use longer sampling times, it might be necessary to use more analog filtering before recording.

Any additional filtering done before processing the data is best done digitally. A satisfactory amount is the equivalent of five more stages of low-pass resistance-capacitance filtering with time constants of 27 sec for three stages and 150 sec for two stages. The differential equation corresponding to a single stage of the above filtering is

\[ y' + wy = wx, \] (1)

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\[ y' + wy = wx, \] (1)
where

\[ x = \text{unfiltered value}, \]
\[ y = \text{filtered value}, \]
\[ w = 1/\text{time constant}, \]

and the prime denotes differentiation with respect to time. This filtering of course introduces different phase lags for different frequencies, but this is of no consequence because all variables are filtered with the same filter. More and better smoothing can be used later in the computation of corrected gravity; but in the crosscorrelation analysis it is desirable to make the computations with only a small amount of filtering.

The filtered monitors will be designated by \( M_1, M_2, \ldots, M_n \), and filtered observed gravity will be designated by \( M_0 \). Filtered corrected gravity will be defined as

\[ G = M_0 + \sum_{i=1}^{N} h_i M_i, \quad (2) \]

where the \( h_i \) are determined by the condition that there should be no crosscorrelation between the curvature of \( G \) and that of any monitor. The curvatures of the monitors and observed gravity are obtained by two differentiations; the curvatures will be designated by \( m_1, m_2, \ldots, m_n \) for the monitors and by \( m_0 \) for observed gravity.

Differentiating equation (2) twice gives

\[ g = m_0 + \sum_{i=1}^{N} h_i m_i, \quad (3) \]

where \( g \) = the curvature of \( G \). If there is zero crosscorrelation between \( g \) and all the monitors, the following equations will be satisfied:

\[ \int g m_j dt = 0, \quad j = 1, \ldots, N, \quad (4) \]

if \( Y_{ij} \) is defined as

\[ Y_{ij} = \int m_i m_j dt, \quad i, j = 0, \ldots, N, \quad (5) \]

and if (3) is substituted into (4), then

\[ Y_{0j} + \sum_{i=1}^{N} h_i Y_{ij} = 0, \quad j \neq 0. \quad (6) \]

The matrix \( Y_{ij} \) is computed from the data and substituted into equation (6) to give simultaneous equations which can be solved to give the \( h_i \). Then \( h_i \) are substituted into equation (2) to give \( G \), the corrected value of gravity.

It is advantageous to compute a normalized matrix \( V_{ij} \) corresponding to \( Y_{ij} \) because it gives some information that is not readily apparent from \( Y_{ij} \). For this normalization, \( k_i \) and \( V_{ij} \) will be denoted as

\[ k_i = h_i \sqrt{Y_{ii}/Y_{00}}, \quad (7) \]

and

\[ V_{ij} = Y_{ij}/\sqrt{(Y_{ii}Y_{jj})}, \quad i, j = 0, \ldots, N. \quad (8) \]

Then equation (6) gives

\[ V_{0j} + \sum_{i=1}^{N} k_i V_{ij} = 0, \quad j \neq 0. \quad (9) \]

Equation (8) shows that \( V_{ij} \) is the desired normalized matrix and that its nondiagonal elements are the usual crosscorrelation coefficients for zero time difference. These coefficients are useful in interpreting the results obtained from actual data.

An estimate of how much the accuracy is improved by making corrections to observed gravity by the crosscorrelation method is the ratio of the curvatures of corrected and observed gravity. Obviously this is only an estimate because some of the curvature of the observed gravity trace is due to actual gravity anomalies and some is probably due to errors in the Eötvös correction, an accurate record of which might not be available as a monitor. To compute the curvatures of observed and corrected gravity, it is noted that their squares are proportional, respectively, to

\[ C_o^2 = Y_{00}, \quad (10) \]

and

\[ C_e^2 = \int g^2 dt. \quad (11) \]

If \( h_0(=k_0) \) is taken equal to 1, it follows from equation (11) that

\[ C_e^2 = \sum_{i=0}^{N} h_i h_j Y_{ij}. \quad (12) \]

If we make use of (6) it can be seen that the ratio of the rms curvature of corrected gravity to that of observed gravity is
Crosscorrelation for Shipboard Gravity

\[ \frac{C_c}{C_0} = \sqrt{\left( \sum_{i=0}^{N} h_i h_j Y_{ij} / Y_{00} \right)} \]

\[ = \sqrt{\left( \sum_{i=0}^{N} h_i Y_{i0} / Y_{00} \right)}. \quad (13) \]

Also using equations (6), (7), (8), and (9), equation (13) can also be written as

\[ \frac{C_c}{C_0} = \sqrt{\left( \sum_{i=0}^{N} k_i k_j V_{ij} \right)} \]

\[ = \sqrt{\left( \sum_{i=0}^{N} k_i V_{i0} \right)}. \quad (14) \]

It can be shown that

\[ (\frac{C_c}{C_0})^2 \leq 1. \quad (15) \]

This result is important because it states that the square of the curvature of corrected gravity can be less than but not greater than that of observed gravity, regardless of how the monitors are chosen.

Because of the importance of this feature of the crosscorrelation method, it is worthwhile to outline the proof of (15). First, it should be noted that the results of applying the method are not affected by using a set of monitors \( P_1, \ldots, P_n \) that are linear combinations of the original monitors \( M_1, \ldots, M_n \). For convenience observed gravity will be denoted by \( P_0 \), and the curvature of any \( P_i \) will be denoted by \( p_i \). The normalized matrix elements corresponding to \( V_{ij} \) will then be

\[ U_{ij} = \int p_i p_j dt / \sqrt{\left( \int p_i^2 dt \right) \cdot \left( \int p_j^2 dt \right)}. \quad (8') \]

If the old monitors are independent, it is always possible to choose an independent set of the new monitors \( P_i \) that are linear combinations of the old monitors and which make

\[ U_{ij} = 0 \quad \text{for} \quad i \neq 0, \quad j \neq 0, \quad \text{and} \quad i \neq j. \]

One way to do this is as follows: Take \( P_1 = M_1 \). Then take \( P_2 = M_2 + a_{12} M_1 \) with \( a_{12} \) chosen to make \( U_{12} = 0 \). Next take \( P_3 = M_3 + a_{13} M_1 + a_{23} M_2 \) with \( a_{13} \) and \( a_{23} \) chosen to make \( U_{13} = U_{23} = 0 \), etc.

If the equation corresponding to (3) is taken as

\[ g = p_0 + \sum_{i=1}^{N} l_i p_i, \quad (3') \]

then the equation corresponding to (9) is

\[ U_{0j} + l_j = 0, \quad j \neq 0, \quad (9') \]

and with \( l_0 = 1 \), the equation corresponding to (14) is

\[ (\frac{C_c}{C_0})^2 = \sum_{0}^{N} l_i U_{0i} \]

\[ = 1 - \sum_{1}^{N} (U_{0i})^2. \quad (14') \]

Since the left side of (14') is a square, it is positive. Also, since all the terms in the summation on the right are squares, the right side is equal to or less than one, and (15) is true.

At this point it is helpful to summarize some of the main features of the preceding mathematical treatment; they are: (a) The basis for making corrections is a comparison of the curvatures of the monitors with that of observed gravity. Therefore, the method cannot be used unless the data show appreciable curvatures in the monitors. Fortunately, shipboard data almost invariably satisfy this requirement. (b) The method gives a corrected gravity whose rms curvature cannot be greater than that of the observed gravity regardless of what monitors are chosen. (c) Corrections are made to observed gravity by adding the fractions of the monitors determined from comparing the curvatures. These corrections are equivalent to extrapolations, and they include systematic corrections. In order to make the extrapolations, the monitors must be chosen so that they all equal zero on a stationary ship. (d) The accuracy of the extrapolations depends on the adequacy of the data available and on the choice of monitors. These two problems will be considered next.

One effect of insufficient data is that it can give high correlation between even well-chosen monitors. This can happen because correlation depends on ship motion as well as on monitors. For a typical example, consider the two monitors \( M_1 = \langle z' \rangle^2 \) and \( M_2 = z' y'' \), where \( z' \) is the vertical velocity and \( y'' \) and \( z'' \) are horizontal and vertical accelerations. On a single line it often happens that \( z', z'', \) and \( y'' \) are roughly proportional to each other and the phase angles between them are...
roughly constant. Under such conditions, there will be strong correlation between \( m_1 \) and \( m_2 \). However, if the direction of travel of the ship is reversed, the phase angle between \( z' \) and \( y'' \) will change approximately 180 degrees, which will give a correlation of the opposite sign. If the data being processed include travel in both directions, there might even be zero correlation.

An effect of high correlation between monitors is that it can lead to ambiguity in the optimum fractions \( h_i \) of the monitors to be used in the computation of corrected gravity. For example, assume the extreme case in which \( M_1 \) and \( M_2 \) are accurately proportional to each other for the limited data available. Then the \( V_{10} \) element of the normalized matrix will equal one, and the simultaneous equations (9) will give a solution only for the sum of \( k_1 \) and \( k_2 \). This means that the same corrected gravity will be computed regardless of whether \( M_1 \), \( M_2 \), or a linear combination of them is used to make the correction.

Let us now consider a slightly different case. In the previous case it was assumed that the two monitors \( M_1 \) and \( M_2 \) were accurately proportional to each other for the limited data available. It will now be assumed that the two curvatures \( m_1 \) and \( m_2 \) are accurately proportional, but the monitors themselves are not. In this case the ambiguity in the determination of the optimum fractions \( h_1 \) and \( h_2 \) of the monitors would lead to an ambiguity in the computation of corrected gravity. The problem could, of course, be solved by making use of enough data to eliminate the ambiguity, but it is also worth investigating how important the error might be if the ambiguity is not removed.

It has previously been stated that the typical reasons for high correlations between monitors are: (1) The amplitudes of the various components of acceleration and velocity are roughly proportional to each other for data taken on only a single line, and (2) the phase angles between the above components are roughly constant for such data. Fortunately these conditions make the monitors roughly proportional to each other when their curvatures are roughly proportional. It will be remembered that this is the condition for getting a good systematic correction for gravity in spite of ambiguity in the relative contributions of the individual monitors. The conclusion that a good gravity correction can generally be made under these conditions is consistent with the results computed from some actual data that are given later.

The choice of monitors will now be considered. The criterion for choosing them is whether they can make significant reductions in the curvature of corrected gravity for likely malfunctions in the gravity meter or stabilized platform. Since the crosscorrelation method is not intended to correct for erratic malfunctions such as the results of a bad bearing in the stabilized platform, it is not necessary to use the higher-degree terms in the power series. The lower-degree terms are capable of detecting but not correcting for erratic malfunctions. The omission of the higher-degree terms reduces the likelihood of getting high correlations between monitors. For example, if \((z'')^4\) were used as a monitor, considerable correlation between it and \((z'')^2\) might be expected for limited amounts of data.

It has been found for LaCoste and Romberg gravity meters that no first-degree terms need be considered except the Eötvös effect. Although the Eötvös effect was not used as a monitor in this article, it could have been used if it had been available. Of the twenty-one possible second-degree terms, only six were used. No third-degree terms have yet been found to give appreciable systematic corrections, but one of them has reduced the curvature of corrected gravity; it is therefore being used. It corresponds to the horizontal direction in which the gravity meter has the least stiffness. Possible ways in which the monitors can produce errors in a spring-type gravity meter have been discussed by the author (LaCoste, 1967). Similar errors can also be produced in a stabilized platform by four of the chosen monitors. As previously mentioned, if the stabilized platform has slow servos, the monitors might also have to include rates of roll and pitch of the ship.

RESULTS FROM ACTUAL DATA

LaCoste and Romberg shipboard gravity meters normally provide data from seven monitors. These monitors are believed to be sufficient for determining how well the gravity meter is performing and for making corrections if the meter is not performing well. Using \( x \) and \( y \) as horizontal coordinates and \( z \) as the vertical, the seven monitors are: \((z'')^2, y/z', y/z'', (x'')^2z'', \) and the absolute values of \( x'' \) and \( y'' \). The crosscorrelation correction method previously described was
Crosscorrelation for Shipboard Gravity

Table 1. Normalized crosscorrelation integrals (crosscorrelation coefficients for zero time differences)

<table>
<thead>
<tr>
<th></th>
<th>y&quot;z'</th>
<th>y&quot;z&quot;</th>
<th>x&quot;z&quot;</th>
<th>(z&quot;)^2</th>
<th>x&quot;</th>
<th>y&quot;</th>
<th>(x'')^2 z&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>.131</td>
<td>-.17</td>
<td>.959</td>
<td>-.152</td>
<td>-.173</td>
<td>-.14</td>
<td>-.119</td>
</tr>
<tr>
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<td>-.117</td>
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<td>-.152</td>
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<td>1</td>
<td>.189</td>
<td>.678</td>
<td>-.062</td>
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<td>-.14</td>
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<td>-.029</td>
<td>.024</td>
<td>1</td>
</tr>
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</table>

applied to about 17 hours of data obtained with a gravity meter that was not operating properly. Table 1 shows the normalized crosscorrelation integrals. The first row of this matrix are the usual crosscorrelation coefficients for zero time difference. The high crosscorrelation coefficient between gravity and x"z" indicates that x"z" is the main source of error. High crosscorrelations are also shown between the following monitors: y"z', y"z", (z")^2, and |y"|. These correlations will be discussed later.

Table 2 gives the fractions of the monitors required to be added to observed gravity in order to get zero crosscorrelation with all monitors. At the bottom of the table is the ratio of the rms curvature of corrected gravity to that of observed gravity. It is 0.26, which indicates considerable improvement in smoothness. Table 3 gives the average of systematic corrections in mgal. As expected, x"z" gives the largest contribution, 4.3 mgal. The total systematic correction is 5.2 mgal. It should also be noted that the sum of the corrections of the four previously mentioned highly correlated monitors is only 0.6 mgal, which indicates their combined effect is small.

Figure 3 is a plot of observed and corrected gravity over a portion of the data; the portion chosen is representative of all the data. The corrected gravity curve shows a striking improvement in smoothness over the observed gravity curve. Since this smoothness was not obtained by additional filtering, it is a good indication of improved accuracy. Both the observed and corrected gravity curves were plotted from data that had been filtered only very slightly: 3 stages of 20-sec, plus 3 stages of 27-sec resistance-capacitance filtering. This small amount of filtering was used to emphasize the results of the method. Furthermore, no Eötvös corrections were used because they were not available to the writer. It

Table 2. Fractions of monitors to be added to gravity to get zero crosscorrelation with all monitors

<table>
<thead>
<tr>
<th>y&quot;z'</th>
<th>y&quot;z&quot;</th>
<th>x&quot;z&quot;</th>
<th>(z&quot;)^2</th>
<th>x&quot;</th>
<th>y&quot;</th>
<th>(x'')^2 z&quot;</th>
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</thead>
<tbody>
<tr>
<td>.275</td>
<td>-.139</td>
<td>-.2481</td>
<td>.21</td>
<td>-.045</td>
<td>.066</td>
<td>.375</td>
</tr>
</tbody>
</table>

(rms curvature of corrected gravity) = 0.261
(rms curvature of observed gravity)
is therefore not surprising that there is some remaining roughness in the corrected gravity curve.

It has previously been mentioned that the processed data show strong correlation between the monitors \( y''z' \), \( y''z'' \), \( z'' \), and \( y'' \). Arguments have also been presented to show that in such cases the data are not adequate to give an accurate determination of the relative optimum amounts of the strongly correlated monitors. However, it has been contended that it is sufficient to give a good overall correction for their combined effect, for the available data. It is possible to check this conclusion for consistency with the available data in the following way. Since the data show strong correlation between the four previously mentioned monitors, three of them can be discarded from the crosscorrelation analysis, and the fourth, \( y''z'' \), can be used to take care of the effects of all four. When this procedure was followed, the corrected gravity curve was found to be almost identical to the previously obtained curve. Deviations from the original corrected gravity curve are shown dotted in Figure 1. It should be noted that even the systematic corrections are almost identical.

Another question worth investigating is, "What correction will be obtained for observed gravity if the monitors chosen have no connection with ship motion?" As an example of monitors that have little connection with ship motion, a set of six sinusoidal functions might be chosen. Using only six such functions will not give a good approximation to ship motion because an adequate Fourier expansion would require many more terms. It is desirable to choose the periods of the sinusoids in the range transmitted by the filters used in the analysis. Accordingly, periods of 3, 6, and 12 minutes were chosen for both sine and cosine components. For a four-hour segment of the previously analyzed data, the rms corrections made by the sinusoidal monitors were all found to be less than 0.05 mgal. Also, the ratio of the curvature of corrected gravity to that of observed gravity was 0.995. These figures show a negligible

Table 3. Average systematic corrections in milligals made to observed gravity

<table>
<thead>
<tr>
<th>( y''z' )</th>
<th>( y''z'' )</th>
<th>( x''z'' )</th>
<th>( (z'')^2 )</th>
<th>( x'' )</th>
<th>( y'' )</th>
<th>( (x'')^2 z'' )</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.4</td>
<td>0.9</td>
<td>4.3</td>
<td>0.5</td>
<td>-0.1</td>
<td>0.4</td>
<td>0.3</td>
<td>5.2</td>
</tr>
</tbody>
</table>

FIG. 3. Comparison of observed and corrected gravity.
Crosscorrelation for Shipboard Gravity

correction, which indicates that monitors related to actual ship motion must in general be used to obtain good results. The correction computed with the sinusoidal monitors can be looked upon as filtering with a filter that removes only the three sharply defined periods used and no other periods. Such a filter would be expected to smooth the data slightly but not significantly.

CONCLUSIONS

The gravity meter which produced the data under consideration has since been returned for repairs. Laboratory tests confirmed the prediction that the main error was associated with the \( x''z'' \) monitor. The laboratory tests indicated an optimum fraction of this monitor of \(-3.2\) compared to the computed value of \(-2.5\), which was obtained several months before the return of the gravity meter. Data taken at sea shortly before the return of the gravity meter gave an optimum fraction of the \( x''z'' \) monitor of \(-3.1\), which is in excellent agreement with the laboratory determined value. It appears that there was a gradual deterioration related to the monitor in question because data taken in the initial operation of the meter at sea gave an optimum fraction of only \(-0.02\).

The second largest error indicated by the cross-correlation analysis is related to the \( (x'')y'' \) monitor. The analysis indicated an optimum fraction of 0.375, while laboratory tests indicated a value of 0.36. In order to determine how important the contributions of the other monitors are, a crosscorrelation analysis was made using only the above two most significant monitors. The corrected gravity curve obtained was not appreciably different from the one obtained using all seven monitors. This result was confirmed by laboratory tests which showed that all adjustments were close except those relating to the two most significant monitors mentioned above. These comparisons indicate not only that the crosscorrelation method of analysis gives reliable results, but also that the laboratory tests give the same results as actual sea tests.

The agreement between the crosscorrelation analysis and laboratory tests shows that it would have been possible to compute the required changes in resistors to restore good gravity meter operation. Also, these resistor changes could easily have been made at sea. The possibility of making such a repair, or being able to correct the data even though the repair is not made, is of considerable importance. It can avoid the time-loss and costs of interrupting a survey as well as the transportation costs for the repair of the gravity meter.

On the other hand, it should be pointed out that a crosscorrelation analysis can not always give a good correction for bad data. For example, if the \( x''y'' \) monitor had been lost in the case under consideration, a good correction would have been impossible. The remaining monitors would have given a partial correction but not a good one. Another example is a bad bearing in the stabilized platform. The behavior of a bad bearing is too erratic to allow the monitors to give a good correction in such a case. However, in either of the above cases, the available monitors would show enough crosscorrelation to indicate poor instrument performance and possibly its source. Even if crosscorrelation is used merely to evaluate gravity meter performance, it can be of considerable use.

Minicomputers are now being used on shipboard gravity meters to compute the crosscorrelation coefficient for any chosen monitor while the system is in operation. This makes possible data evaluation on the spot. Complete crosscorrelation analyses at sea have not yet been made, but they can be made if it appears worthwhile.

Needless to say, the gravity meter that gave the bad results was carefully checked for any flaws in its construction. It was found that a wire clamp in it had not been adequately tightened because one of the clamping screws was too long. The screw had bottomed before it had properly tightened the clamp. It is believed that slight slipping of the wire in the clamp had changed the gravity meter adjustment thereby introducing the errors. After repair, the gravity meter was again sent to sea and after several months has been returned. A second set of tests showed that the gravity meter characteristics had not changed since the repair had been made.

REFERENCES