GEOPHYSICS, VOL. 42, NO. 3 (APRIL 1977); P. 594–601, 5 FIGS., 1 TABLE

1974 TEST OF LACOSTE AND ROMBERG INERTIAL NAVIGATION SYSTEM

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In mid-1974 a test was made in the Gulf of Mexico of a LaCoste and Romberg inertial system for the measurement of the Eötvös correction for shipboard gravity meters. Since the system is designed for operation with satellite navigation, inertial velocities were updated at 2- to 3-hour intervals, using Lorac information because no satellite information was available. Two types of comparison were made between the inertial and Lorac data. One comparison was the rms difference between results from the two methods—0.46 knot, which corresponds to 3.0 mgal at a latitude of 30 degrees. The other comparison related to noise which could be mistaken for anomalies of interest in oil exploration. The comparison indicated that such noise in the inertial data was only about one third that in the Lorac data.

INTRODUCTION

From June 30 to July 7, 1974, tests were made with LaCoste and Romberg shipboard gravity meter S-28 which had been modified to include the LaCoste and Romberg inertial navigation system. The modified gravity meter is owned by the U.S. Naval Oceanographic Office and was lent for the test. It was operated by LaCoste and Romberg personnel on board an Edcon ship, which was doing a routine gravity survey with another LaCoste and Romberg gravity meter in the Gulf of Mexico. Edcon provided Lorac information for comparison with the inertial data.

The instrument used in the test was essentially the same as that described by Valliant and LaCoste (1976). Features that were the same include:

1) Honeywell GG49 gyros were used.
2) The horizontal and azimuth stabilization loops all included damping to avoid resonant oscillations.
3) Computations were made solely from gyro and accelerometer outputs as though the system were a strap-down system.

Features of the inertial system that were different from the one described in the reference were:

1) The horizontal stabilization loops had periods of 17 minutes instead of 4 or 12 minutes to reduce gyro precession rates.
2) Data were taken with azimuth stabilization loop periods of both 84 minutes and 6 hours.

DATA PROCESSING

The equations used in data processing are given in my article with Valliant (1976), which I will subsequently refer to as our previous inertial article. The approximate equation (21) of that article was used in my present computations; it can be written as

$$V'' + W_s^2 V = A' + g\phi', \qquad (1)$$

where

- V = eastward speed including the speed due to rotation of the earth in inertial space,
- W_s = 2π/Schuler period = .00124 per sec,
- A = east accelerometer output,
- ϕ = output of the gyro whose sensitive axis is north,

and the (′) indicates differentiation with respect to time. Since the above equation refers to a coordinate system in which the azimuth is perfectly

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stabilized in the north direction, a correction was made for azimuth variations resulting from gyrocompassing. The correction was an azimuth axis transformation using the azimuth obtained by integrating the sum of the azimuth gyro rate plus an appropriate constant. The constant was necessary to take into account the effect of gyro drift and earth rotation. It was chosen to make the azimuth equal to zero at the start and end of a computation interval.

As explained in our previous inertial article, equation (1) is the differential equation for an undamped harmonic oscillator, and therefore the initial conditions for its solution must be chosen correctly to avoid 84-minute sinusoidal oscillations (Schuler oscillations) in the solution. The previous article suggested three different criteria for choosing the initial conditions: (a) Remove all 84-minute oscillations in the Eötvös correction for time intervals during which the ship is thought to have been traveling in a straight line at constant speed; (b) make the 84-minute oscillations in inertial Eötvös the same as for observed gravity, or (c) make the 84-minute oscillations in inertial Eötvös the same as those for Eötvös computed from dead reckoning. Although method (c) is generally the best of the three methods, it was not used because the ship heading data for dead reckoning had too much electronic noise. Instead, method (a) was used.

All computations were made by post-data-processing rather than in real time in order to make it possible for the computer program to recognize and remove Schuler oscillations from the results. Obviously, this removal of Schuler oscillations by post-data-processing greatly improves the accuracy.

As noted in the previous article, the LaCoste and Romberg inertial system was designed for use with satellite navigation or some other ancillary means of navigation that can provide velocity updates at 2- to 3-hour intervals. The updating procedure adds whatever constant is necessary to the inertial velocity to make its average value over the interval the same as the average value determined by the ancillary means. In the present case, updates were made from Lorac data at the beginning and end of each line. The durations of the lines were generally 2 to 3 hours.

The Eötvös equation given in my previous article is equivalent to

$$E = V^2/R + R(\lambda')^2 - RW^2\cos^2\lambda \qquad (2)$$

where

E = Eötvös correction,
R = radius of the earth,
λ = latitude, and
W = rotational rate of the earth.

Equation (2) reduces to the usual Eötvös equation by substituting

$$V = V_0 + v_E, \text{ and}$$
$$v_N = R\lambda',$$

where $V_0 = RW\cos\lambda$ = the eastward speed due to rotation of the earth in inertial space, and v_E and v_N = respectively, the east and north speeds of the ship relative to the earth. The substitution gives the Eötvös correction as

$$E = (2V_0v_E + v_E^2 + v_N^2)/R.$$

The reason for using (2) in its original form in inertial computations is that (1) gives V directly rather than v_E.

In using (2) to compute the inertial Eötvös correction, two approximations were made. One approximation was to neglect the second term on the right. This approximation neglects the effects of the northward speed of the ship; it is an approximation that is very often made in shipboard gravity work. The other approximation was to neglect the last term on the right of (2). This approximation neglects the effects of latitude changes on the earth's centripetal acceleration. The approximation gives no errors on east-west courses because they involve no latitude changes. Fortunately, nearly all of the courses were east-west. However on north-south lines, the approximation gives a small monotonically increasing or decreasing error along the course. Without this error on north-south lines, the comparison between inertial and Lorac Eötvös might have been slightly better. The reason for having to make the latter approximation was that Edcon had furnished us Lorac Eötvös data but no detailed latitude information. I did not request any further information because of the confidentiality of the survey and because of the effort involved in their looking through old data. It would have been possible to determine latitude changes inertially if occasional satellite fixes had been available.

The data recorded from the test included essentially raw (very slightly filtered) data as well as some gravity data which had been processed and filtered by an on-line minicomputer. The on-line processed data were found to have errors from

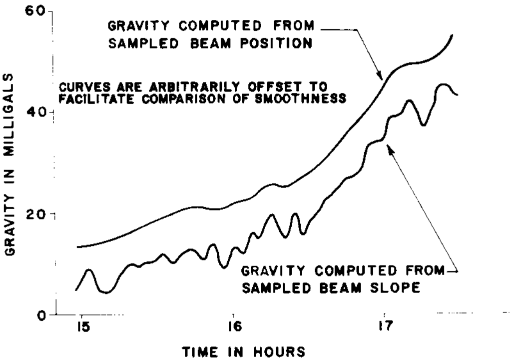

FIG. 1. Aliasing errors resulting from differentiating before sampling data.

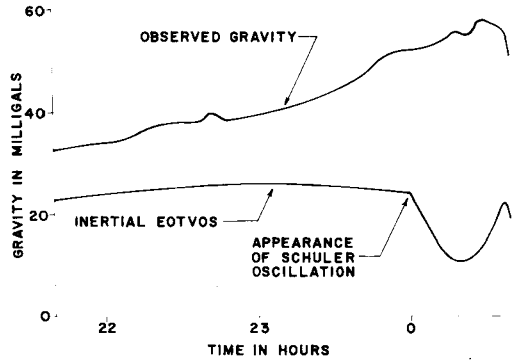

FIG. 2. Inertial Eötvös for line 333 showing appearance of Schuler oscillation near end of record.

two sources, namely, occasional digital transmission errors to the minicomputer and aliasing errors in the rougher weather. It was possible to eliminate both types of error by recomputing from the raw data. All the large digital transmission errors were found by a simple computer program and were corrected by interpolation; there were only five of them. The computer program for detecting and correcting such errors is now being incorporated on all on-line minicomputers used with LaCoste and Romberg gravity meters.

The way in which the aliasing errors occurred in rough weather is as follows. In LaCoste and Romberg gravity meters, gravity is computed as the sum of the vertical force exerted by the gravity meter spring and movable beam plus a constant times the velocity of the beam. The aliasing error occurred because the velocity of the beam was not sampled often enough in rough weather. However, it was found that the errors could be eliminated without shortening the sampling time by using the beam position rather than its derivative in computing gravity. (Beam position data are routinely recorded on magnetic tape.) The beam position is so much less noisy than its derivative that the 10-sec sampling time was more than adequate. Differentiation of the beam position signal was, of course, done in the computer. Figure 1 shows a comparison of gravity computed by the two methods for the roughest line in the test. The curves were given arbitrary offsets to facilitate the comparison of smoothness. The improvement is striking for this line although most of the lines showed no improvement. The new method of computation is now being used in all on-line minicomputers used with our shipboard gravity meters. It should be pointed out, however, that the aliasing error just described has not occurred in the past on our shipboard gravity meters except when on-line minicomputers were used with them or when digital records of the slightly filtered beam velocity were used to recompute gravity. In the great majority of the installations, no on-line minicomputers have been used, possibly because of the problem just described. When no on-line computer is used, gravity is computed by an analog computer which has performed well. Although the digital sampling time of the analog computer output is 10 sec, this sampling time is adequate because the analog computer output is filtered more than the beam velocity data previously mentioned.

RESULTS

Two types of comparison are made between inertial and Lorac Eötvös. The first comparison is the difference between the two types of Eötvös; the second is a comparison of the accuracy of detail. Table 1 gives the results of the test for each line and for a combination of all lines. As previously mentioned, the time duration of each line is the update interval for that line. It is given in column 2, and is generally 2 to 3 hours. The rms differences between inertial and Lorac Eötvös are given in column 3, which also gives the rms difference for all the lines as 3.0 mgal. Since the test was made at a latitude of about 30 degrees, this rms difference corresponds to 0.46 knot.

An inspection of Table 1 shows that the rms difference between inertial and Lorac Eötvös is abnormally large for line 333. The reason for this large value is indicated in Figure 2, which shows inertial Eötvös for the line in question. It can be

Table 1. Comparison of Lorac and inertial Eötvös.

Line	Duration	Lorac-inertial difference	rms variations of		rms variations of gravity with		
			Lorac Eötvös	Inertial Eötvös	No Eötvös	Lorac Eötvös	Inertial Eötvös
	minute	mgal	mgal/(minute)^2				
100	168	1.6	.106	.050	.184	.213	.193
101	182	1.1	.061	.061	.165	.173	.178
102	166	1.4	.191	.050	.090	.219	.076
103	75	2.4	.199	.054	.147	.278	.135
210	183	2.9	.461	.155	.201	.533	.187
220	169	3.3	.193	.064	.109	.219	.08[illegible]
221	93	2.3	.162	.146	.299	.319	.285
230	170	1.3	.079	.065	.147	.156	.138
240	137	1.5	.135	.070	.165	.193	.149
300	87	2.0	.129	.052	.100	.155	.081
301	144	2.8	.118	.074	.138	.191	.151
310	139	1.0	.083	.065	.279	.280	.267
320	151	5.5	.091	.088	.284	.267	.269
321	153	1.1	.073	.119	.092	.094	.127
322	137	5.2	.075	.140	.259	.276	.288
323	149	2.4	.136	.078	.099	.159	.105
331	135	1.1	.174	.062	.116	.212	.139
332	138	.9	.090	.062	.116	.135	.127
333	122	11.5	.182	.199	.242	.287	.317
400	141	1.1	.084	.049	.091	.121	.087
401	133	1.8	.078	.077	.179	.192	.203
402	145	2.8	.174	.105	.126	.198	.125
411	131	1.5	.099	.059	.146	.154	.140
420	141	2.8	.222	.069	.123	.225	.089
421	129	3.7	.083	.036	.432	.448	.440
431	138	2.4	.080	.038	.168	.191	.161
432	122	2.0	.079	.036	.112	.152	.132
500	132	2.3	.064	.037	.221	.218	.226
501	138	2.1	.240	.039	.133	.295	.128
502	134	2.3	.466	.055	.101	.480	.108
511	139	1.1	.254	.051	.437	.499	.439
512	130	2.7	.194	.043	.114	.214	.123
513	135	1.1	.168	.040	.113	.202	.109
521	131	2.2	.170	.031	.096	.205	.098
522	128	1.6	.065	.042	.209	.214	.206
All	4845	3.0	.181	.080	.191	.262	.195
All except 333	4723	2.4	.181	.074	.190	.261	.190

seen that a Schuler oscillation began near the end of the record. The difficulty is that the oscillation occurred so close to the end of the record that the computer program was unable to recognize and correct it. If data had been available after the end of the line, the computer would have recognized the oscillation, but unfortunately the magnetic tape recording system was stopped too soon. This example shows the need for having data both before and after the time for which computations are being made, which is impossible in real time operation. In view of the above discussion, it can be argued that line 333 should be discarded because the recording system was stopped prematurely. If line 333 is discarded, the rms difference between inertial and Lorac Eötvös is reduced to 2.4 mgal.

In order to compare the accuracy of detail in inertial and Lorac Eötvös, a criterion had to be chosen for making the comparison. It was decided to use the smoothness of the gravity curves corrected with the two types of Eötvös. Smoothness appears to be a good criterion because any errors in the Eötvös will, in general, add to the roughness of the corrected gravity curves. However, it is necessary to define smoothness in a quantitative way to get meaningful results; this was done as follows.

When shipboard gravity meters are used in oil exploration, the gravity anomalies of most interest have time durations of about 5 minutes or more. Therefore, the detail in the Eötvös corrections should be good enough to avoid errors of this duration. Accordingly, it was thought that a

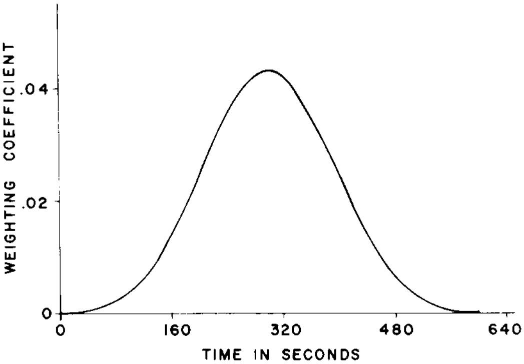

FIG. 3. Filter weighting coefficients used on computations.

good measure of smoothness would be to use the rms variations in the second time derivative of the filtered gravity curves. The second derivative was computed from the expression

$$d^2G(t)/dt^2 = [G(t + 2) - 2G(t) + G(t - 2)]/4,$$

where $G(t)$ = the value of gravity in milligals at the time t minutes. The result appears in units of milligals/(minute squared). Gravity was filtered by the standard L & R digital filter whose weighting coefficients are shown in Figure 3.

In order to see how realistic it is to use the above method of measuring smoothness, the method was used to compute the rms variations of the filtered second derivatives of anomalies of different durations. An anomaly with a 1-cosine shape was chosen; it is shown in Figure 4. The results of the computation are shown in the graph in the same figure. The graph has a maximum response for an anomaly duration of about 7.5 minutes, which is not far from the desired duration of about 5 minutes. Since a maximum response at 7.5 minutes rather than 5 minutes probably gives a slight advantage to Lorac, it was decided to use it in the analysis.

The results of the analysis are given in columns 6, 7, and 8 of Table 1; they show, respectively, the rms variations of the second derivatives of observed gravity, gravity corrected with Lorac Eötvös, and gravity corrected with inertial Eötvös. All variables were filtered with the weighting coefficients shown in Figure 3. It can be seen that about 75 percent of the lines show smaller rms variations for inertial Eötvös than for Lorac Eötvös. Also, the rms values for a combination of all the lines shows a reduction in variation of about 23 percent by using inertial rather than Lorac Eötvös.

To get an idea of what the variations in columns 6, 7, and 8 of Table 1 mean, reference should be made to Figure 5. This figure shows gravity curves for line 210 obtained with (1) no Eötvös correction, (2) Lorac Eötvös correction, and (3) inertial Eötvös correction. The three curves are given arbitrary offsets in order to make it easier to compare their smoothness. Line 210 was chosen because it showed the greatest improvement for inertial Eötvös. The improvement (+ or −) in other lines can be estimated by comparing the rms variations in columns 7 and 8 in Table 1.

A study of the data that have been obtained indicates that it it possible to make a more thorough analysis than has been made up to this point. The fundamental equations for further

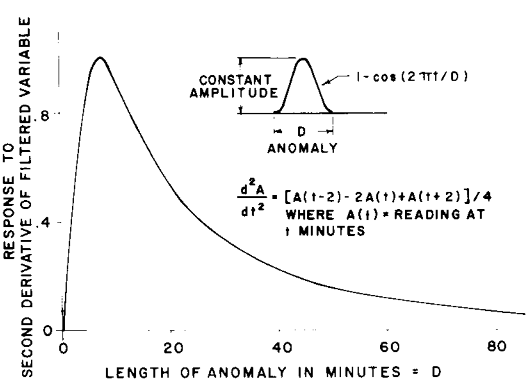

FIG. 4. Response of computer program to filtered second derivative of $1 - \cos(2\pi t/D)$ anomaly.

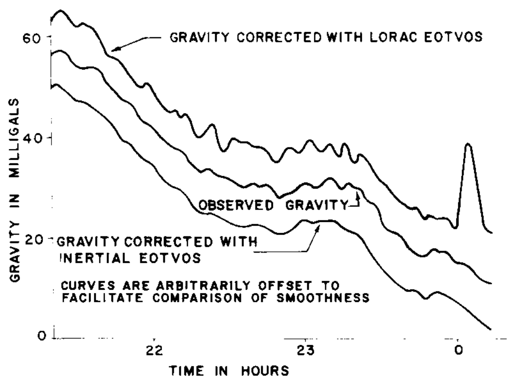

FIG. 5. Comparison of observed gravity, Lorac corrected gravity, and inertial corrected gravity for line 210.

analysis are obtained as follows. We will consider rms variations of the second derivative of each of the filtered variables. (The treatment would be the same for rms variations of any other function of the variables.) We will use the letter i to designate the method (Lorac or inertial) of measuring Eötvös, and we will define

G = the rms variation of the raw gravity meter readings,
E_i = the rms variation of the Eötvös readings made by measurement i,
C_i = the rms variation of the corrected value of gravity obtained by using the Eötvös measurement of method i,
g = the rms variation of the combination of true gravity and gravity meter errors,
e = the rms variation of true Eötvös, and
n_i = the rms variation of the errors in Eötvös measurement by method i.

We will assume that g, e, and n_i are uncorrelated. Then

$$G^2 = g^2 + e^2, \quad (3)$$

and

$$E_i^2 = e^2 + n_i^2. \quad (4)$$

The usual corrected value of gravity is obtained by adding the measured Eötvös to the raw gravity meter reading. The rms variation C_i of the corrected gravity will then include the uncorrelated variations of: (1) true gravity, (2) gravity meter errors, and (3) errors in Eötvös. However C_i will not include the variations in true Eötvös because they are removed in the computation of corrected gravity as described above. We then have

$$C_i^2 = g^2 + n_i^2. \quad (5)$$

Since data are available for G, E_i, and C_i, it is possible to solve equations (3) to (5) for g^2, e^2, and n_i^2. The solutions are

$$g^2 = (C_i^2 - E_i^2 + G^2)/2, \quad (6)$$

$$e^2 = (-C_i^2 + E_i^2 + G^2)/2, \quad (7)$$

and

$$n_i^2 = (C_i^2 + E_i^2 - G^2)/2. \quad (8)$$

In order to use the preceding equations, it was necessary to compute the rms variations of the second derivatives of filtered Lorac and inertial Eötvös. They are given in columns 4 and 5 of Table 1.

Using the combined results at the bottom of Table 1 and equations (6) to (8), the following results are obtained:

For all lines:

	rms variations mgal/(minute)^2	
	Lorac	Inertial
Gravity + errors	.269	.261
True Eötvös	.024	.070
Eötvös errors	.255	.089
Lorac errors/Inertial errors	2.87	

For all lines except 333:

	rms variations mgal/(minute)^2	
	Lorac	Inertial
Gravity + errors	.267	.258
True Eötvös	.027	.074
Eötvös errors	.255	.074
Lorac errors/Inertial errors	3.45	

The above results indicate that the Lorac errors are about three times larger than the inertial errors for the anomalies under consideration.

In order to get an idea how much confidence can be placed in the quantitative aspects of the above results, two criteria will be considered. One criterion is a check between the values of "gravity + errors" obtained by using the two types of Eötvös measurement. The two values should be equal because they should be independent of the method of making the Eötvös measurement. They are seen to be approximately equal.

The second criterion is a check between the two values of "true Eötvös" obtained by using the two types of Eötvös measurement. Here the comparison is not nearly as good. Possible reasons for the disagreement are as follows:

1) The amount of data might be insufficient for a good statistical analysis.
2) Equation (7) gives the true Eötvös as the difference between two relatively large and nearly equal numbers. Therefore, the accuracy cannot be expected to be as good as for the other results.
3) If there are any errors in the factors used in Eötvös computations, an Eötvös error is intro-

duced which is proportional to the Eötvös. Such an error is, of course, correlated with the true Eötvös, which violates a basic assumption of the analysis.

4) A dead band in one of the devices used to measure Eötvös will also introduce an error which is correlated with true Eötvös. This statement can be seen to be true by considering the following example. Assume that the dead band in measurement is so large that the Eötvös measurement gives a constant value over the entire line. Then the Eötvös error will be the negative of the true Eötvös.

DISCUSSION OF RESULTS

Table 1 gives an rms difference between inertial and Lorac Eötvös of 3.0 or 2.4 mgal, depending upon whether line 333 is discarded. Since the Eötvös correction depends upon latitude, it is desirable to express the above differences as velocities. They are 0.46 and 0.37 knots. A comparison of these results with similar results from our previous inertial article might be instructive. Although the previous article did not present the results in a form suitable for the comparison, a comparison was made possible by recomputing from the raw data. The recomputation showed that the difference between inertial and Hi-Fix velocities is 0.61 knots for the 1973 tests, which is not quite as good as the above results for the 1974 tests.

The apparent improvement in the later results might (or might not) be significant. Some difference in results can be expected because of differences in performance of individual gyros and accelerometers. However, the longer periods used in the stabilization loops in 1974 could be the reason for the improvement. A comparison of that portion of the 1974 results taken with the 6-hour azimuth period did appear to be slightly better than that portion taken with the shorter period. The 1973 tests differed from the 1974 tests in being made under open sea conditions in the North Atlantic rather than in the Gulf of Mexico, but it is doubtful whether this is a significant difference.

The most striking result of the test concerns noise that can be mistaken for anomalies of interest in exploration for oil by means of shipboard gravity meters. The test indicates that inertial Eötvös has only about one-third as much such noise as Lorac Eötvös does.

Another way of comparing Lorac with inertial Eötvös is to compare gravity errors at the intersection of a gravity net. I could not make this comparison because of the proprietary nature of the gravity survey. Such a comparison is desirable, but it should be noted that it is different from the noise comparison made in this article. The difference in the two methods can be seen by considering the case in which one type of Eötvös measurement has large short period errors while the other type has large long period errors. The first type of Eötvös would give smaller intersection differences, but the second type would give better profiles and less noise that could be mistaken for anomalies. For detecting very small anomalies, profiles are probably the best means.

In case both Lorac and inertial Eötvös are available in a survey, it is possible to realize the best features of both methods. It might appear that this goal could be achieved by making more frequent inertial updates from the Lorac data. However, this procedure would give discontinuities in the computed Eötvös at the updating times, which would degrade the profiles. This difficulty can be avoided by using the following procedure instead of the usual updating method. We will define

$$C = \text{Lorac Eötvös minus inertial Eötvös.} \quad (9)$$

Next, filter C with a low-pass filter whose break point is at the frequency at which the two Eötvös methods have equal accuracy. Then, using the correct time shift, add the filtered value of C to the unfiltered inertial Eötvös. This procedure makes a negligible correction to the inertial Eötvös at the short periods where it is accurate. However, at long periods, the procedure corrects the inertial Eötvös to the Lorac value. The preceding method is an extension of a method that has been used in which the electronic Eötvös is filtered more than gravity is filtered. The old method is equivalent to using zero Eötvös correction in place of the inertial Eötvös correction in the method I am suggesting. The old method has improved results in many cases.

POSSIBLE FUTURE IMPROVEMENTS

Obviously, the accuracy of the inertial system can be upgraded by using better accelerometers and gyros, but at an increase in cost. Experiments are now in progress with both items. The better accelerometers being used are not very costly, and there is reason to believe that they will give better accuracy. The better accuracy might be expected because the present test indicates that some of the

Schuler oscillations that appeared in processing the data might have been caused by step errors in the accelerometer outputs. The better gyro being tested sells for about $35,000 and, therefore, might be too expensive to use.

A less expensive way to handle the gyro problem is to use duplicate gyros in the direction needed for Eötvös measurement. The use of duplicate gyros can be expected, from statistical considerations, to reduce errors to 0.7 of their previous value. However, the following consideration indicates that a greater improvement can be expected. Since gyro errors are always accompanied by Schuler oscillations in the computed results, it is possible to determine which of the two gyros made the error unless both gyros err simultaneously.

A third improvement that is being tested is the use of 84-minute periods in the horizontal stabilization loops of the system instead of the previous 17-minute periods. Although 84-minute periods are not necessary when the present data processing method is used, they do improve the gyro performance by reducing the precession rate. Furthermore the 84-minute periods keep the platform more nearly level and, therefore, reduce interactions between rotations about the three axes. A better stabilized platform also reduces gravity meter errors on turns, which could be a worthwhile added benefit.

REFERENCE

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